

Maneuvering Simulations for Ships and Sailing Yachts using FRIENDSHIP-Equilibrium as an Open Modular Workbench

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Abstract

An open modular workbench called FRIENDSHIP-Equilibrium has been developed for the analysis of stationary and instationary modes of motion. FRIENDSHIP-Equilibrium offers an integration environment and suitable algorithms for determining the equilibrium of forces and moments acting on ships and yachts, following a force model approach. In the stationary mode the steady state properties of the vessel are determined (hydrostatics, VPP) while in the instationary mode the accelerated rigid body motions are computed (maneuvering motions). The equations of motions may be solved by means of various time integration methods one of which being an advanced time-step adapting Runge-Kutta algorithm. The functionality comprises the computation of added mass and damping as needed for time simulations. Special consideration is given to the aero- and hydrodynamics of sailing. The program therefore also constitutes a refined Velocity Prediction Program. Within the paper various maneuvering situations are presented for both a conventional cargo ship and a contemporary IMS yacht. The examples serve to show the applicability of the approach and to illustrate the usage of the tool. Attention will also be given to IT integration.

1 Introduction

The importance of the performance prediction of ships and sailing yachts at an early design stage increases significantly these days. Besides the prediction of the maximum velocity in a balanced condition the analysis of the ship motions in unsteady conditions receives growing attention. Of major concern are the evaluation of the maneuverability for ships and the speed loss while changing courses for racing yachts.

The FRIENDSHIP-Equilibrium has been developed for both velocity predictions and maneuvering simulations. The forces acting on a ship or sailing yacht are calculated via specific force modules of the software system. The FRIENDSHIP-Equilibrium represents a workbench in which each acting force is taken into account by an individual module. Using the stationary mode steady state calculations are performed by balancing the hydrodynamic, aerodynamic, buoyant and gravitational forces. In the maneuvering mode the equations of motions are integrated to record instationary behavior. The state model comprises linearized equations of motions in six degrees of freedom. The dynamic forces, dependent on the velocity and the acceleration, are considered via the instationary terms in the equations of motion. The instationary mode therefore enables a simulation of motions, especially maneuvering, in which the different state variables can be regarded as functions of time. Hence, with the maneuvering simulations the behavior of the vessel in unsteady conditions can be assessed and improved before the first sea trials commence. Naturally, the quality of the simulation depends on the validity and accuracy of the available force modules.

Within the paper first the software system itself will be presented. Subsequently, the mathematical background will be outlined. Following this, two examples for maneuvering simulations using the instationary mode will be given. The first example will be the simulation of a test maneuver for the Mariner ship which was described in detail by Wolff (1981). The other example will outline the application of the maneuvering mode for a 10m IMS sailing yacht.

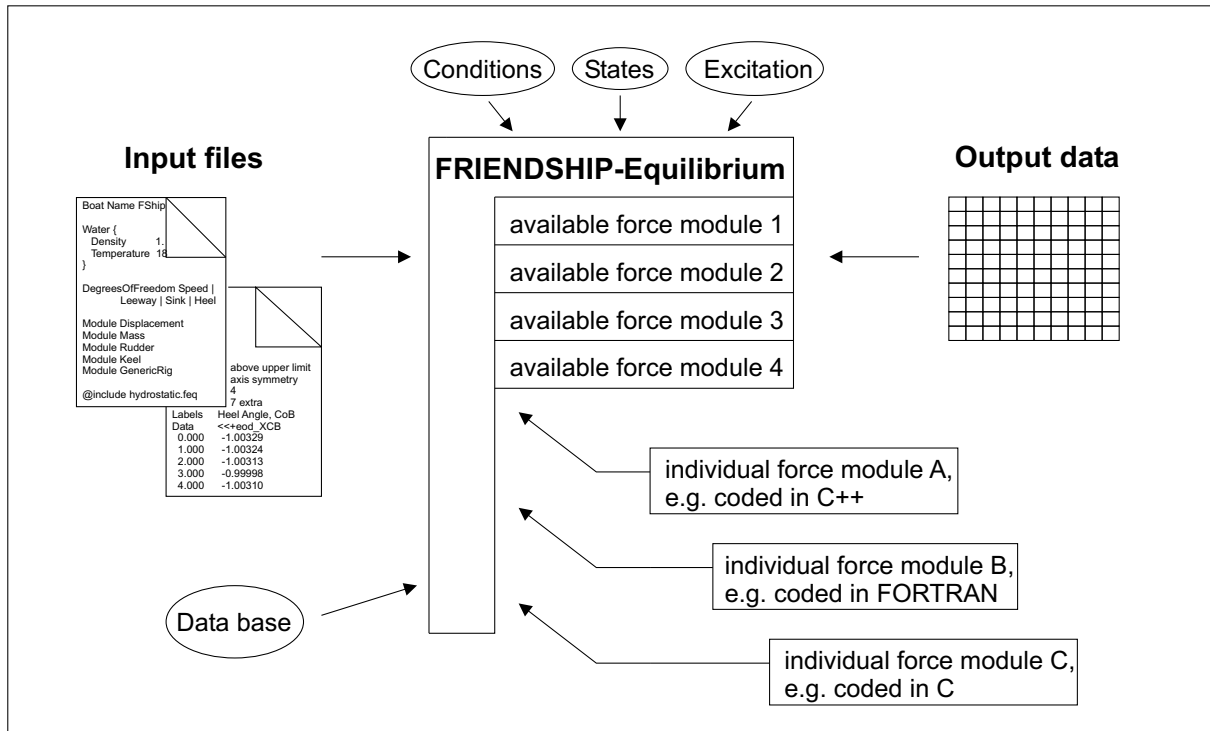


Fig. 1: Open modular structure of FRIENDSHIP-Equilibrium

2 Software system – FRIENDSHIP-Equilibrium

FRIENDSHIP-Equilibrium is an advanced workbench for the analysis of stationary and instationary modes of motion of both ships and sailing yachts. The external forces acting on a vessel for a given state are calculated via various force modules. Each force type like buoyant forces, gravitational forces, rudder forces, keel forces, hull resistance, aerodynamic forces, added resistance in waves, windage etc. is calculated in specific modules. All forces are added up by the program to determine the resulting forces on the vessel.

These modules can be added to and taken out from the simulations individually depending on the design task at hand. A wide selection of force modules already is available. In addition any kind of force acting on the vessel can be defined in a high level language such as FORTRAN, C or C++ and introduced at run time via individual modules, see Fig. 1. The modular structure thus allows for the integration of user provided modules. A prominent application for such a user specific adaptation is the Real-Time VPP at the Twisted Flow Wind Tunnel (TFWT) in Auckland where a force module links the aerodynamic forces measured by the wind tunnel's balance to the program, see *Hansen et al. (2003)*.

Fig. 2 shows an example window of FRIENDSHIP-Equilibrium when used for a velocity prediction of a sailing yacht. The added modules are declared in the 'Input Modules' window. Active modules are marked. Forces which should not be summed up can be declared non-active. The states to be calculated are defined in the 'Cycle Range' window. For the given cycle range the equilibrium conditions are output in the 'Velocity Prediction' table. Below this table the forces of all modules are displayed for the selected state.

For all calculations the desired degree of freedom can be chosen by the user. Up to all six degrees of freedom may be considered in the simulations. The link between the degrees of freedom and the free variables is summarized in Table I. The applications of FRIENDSHIP-Equilibrium for typical scenarios are shown in Table II. The free variables used within these operations are marked with 'X'. The required variables for hydrostatic calculations are indicated as 'input'.

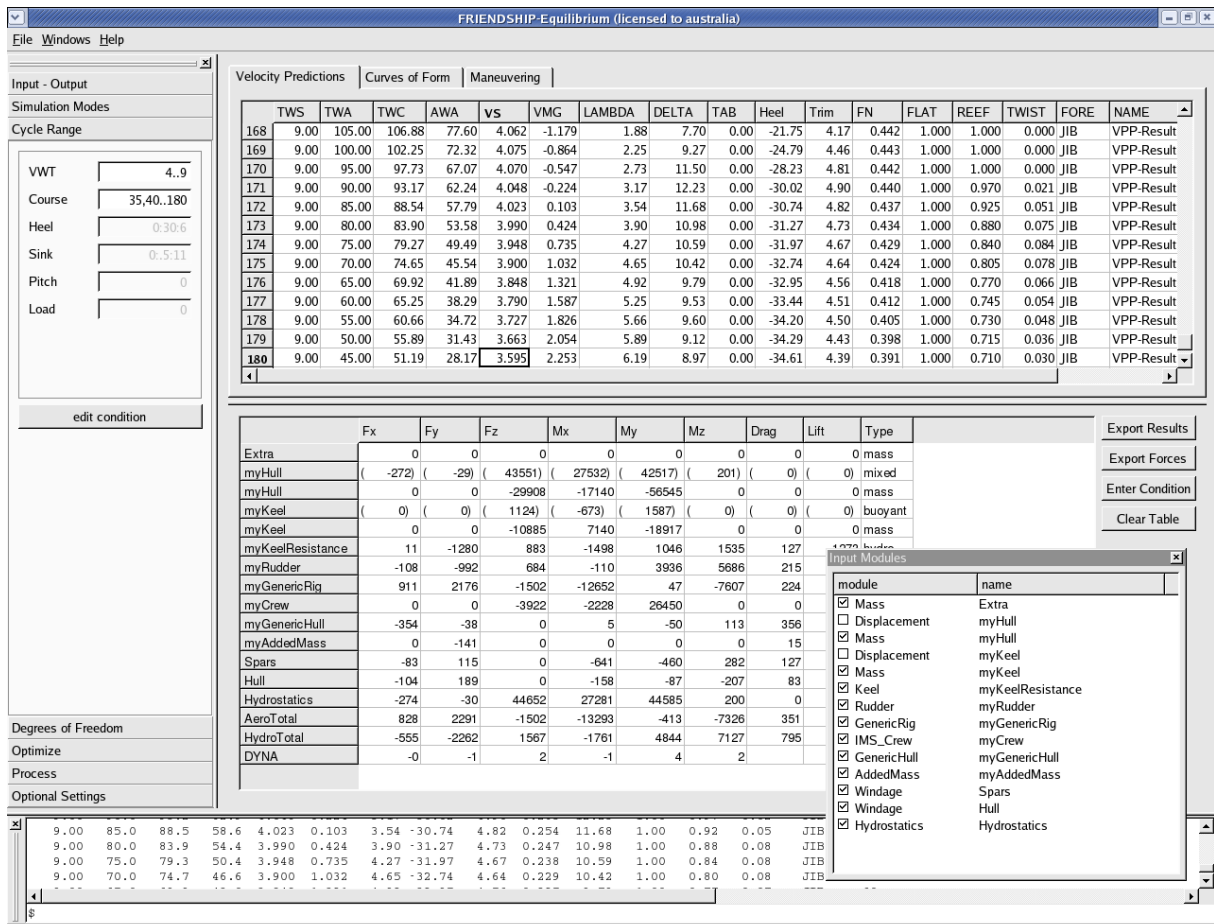


Fig. 2: Modules used in FRIENDSHIP-Equilibrium

Fig. 3 illustrates the structure of FRIENDSHIP-Equilibrium. The used modules and the required parameters are defined in one or several input files, see also Fig. 1. The acting forces are determined for specified environmental conditions. Depending on the force modules these conditions are part of the required input. For instance the force calculation on sailing yachts requires the wind speed and the wind direction as input.

Three simulation modes are offered by the program for different applications:

- stationary mode
- hydrostatic mode
- instationary mode

2.1 Stationary mode

In the stationary mode the steady state of the ship will be determined for specified environmental conditions. The program will resolve the equilibrium in which the sum of all forces add up to zero in the defined degrees of freedom by a nonlinear equation solver. The balance will be computed by means of a Newton-Raphson algorithm. Velocity predictions in steady conditions may also be calculated within this mode. Depending on the considered degrees of freedom, the output contains the state variables displayed in Table I.

Table I: Possible degrees of freedom and constraints for the steady state predictions of a sailing yacht

| Free variable | Condition | | | Comment |
|---------------|------------|----------------|----------------|--|
| | Symbol | Name | Equilibrium | |
| 1 | V_S | ship speed | $\sum F_X = 0$ | basic forces along the centerline, e.g. resistance and aerodynamics, propeller thrust |
| 2 | φ | heel angle | $\sum M_X = 0$ | heeling and righting moments |
| 3 | λ | leeway angle | $\sum F_Y = 0$ | aerodynamic side force to be compensated by hydrodynamic components |
| 4 | δ_r | rudder angle | $\sum M_Z = 0$ | yawing moment to be equalized |
| 5 | δ_t | trim tab angle | $\sum F_Y = 0$ | trim angle to be set to a value giving optimum speed |
| 6 | θ | pitch | $\sum M_Y = 0$ | trimming moments to be compensated |
| 7 | dz | sinkage | $\sum F_Z = 0$ | vertical forces – especially by adding loads as additional cargo, crew, gear or water ballast – as well as dynamic effects to be compensated by additional sinkage |

Table II: Typical scenarios in FRIENDSHIP-Equilibrium (free variables are marked with 'X')

| Scenario | Mode | V_S | λ | dz | φ | θ | δ_r | δ_t |
|-------------------------------------|--------------|-------|-----------|-------|-----------|----------|------------|------------|
| Hydrostatics | hydrostatic | | | X | input | X | | |
| Curves of form | hydrostatic | | | input | input | input | | |
| IMS type standard sailing yacht VPP | stationary | X | (X) | | X | | | |
| 4 DOF sailing yacht VPP | stationary | X | X | | X | | X | |
| Advanced sailing yacht VPP | stationary | X | X | X | X | X | X | X |
| VPP self propulsion | stationary | X | | X | | X | | |
| Maneuvering | instationary | X | X | X | X | X | X | X |

In addition, an arbitrarily set of trim parameters may be defined in the force modules. These parameters are optimized to achieve the maximum speed. Various optimizations routines are available within the program and may be selected by the user. Consequently, desired parameters must be implemented in the modules. The usage of trim parameters is shown in the second example, see section 4.2.

2.2 Hydrostatic mode

The floating position will be calculated depending on the heel angle in the hydrostatic mode. In this mode the equilibria for vertical, roll and pitch movements are resolved. Furthermore, curves of form are determined.

2.3 Instationary mode

For the analysis of motions the program offers an instationary mode. For instationary analysis the excitation forces have to become part of the input. Maneuvering simulations for instance operate with changing rudder angles. Fixed rudder angles can be set. The rudder angles may also be controlled by the use of predefined maneuvers. A PID-controller (Proportional Integral Derivative) is implemented as autopilot to keep a desired course. An additional manual maneuvering module offers the possibility to steer the boat interactively with a joystick. Either way, steering changes are accounted for while running the process.

The motions are described via the equations of motions including all external forces, calculated by their respective force modules. Additional added mass and damping forces are determined via a further force

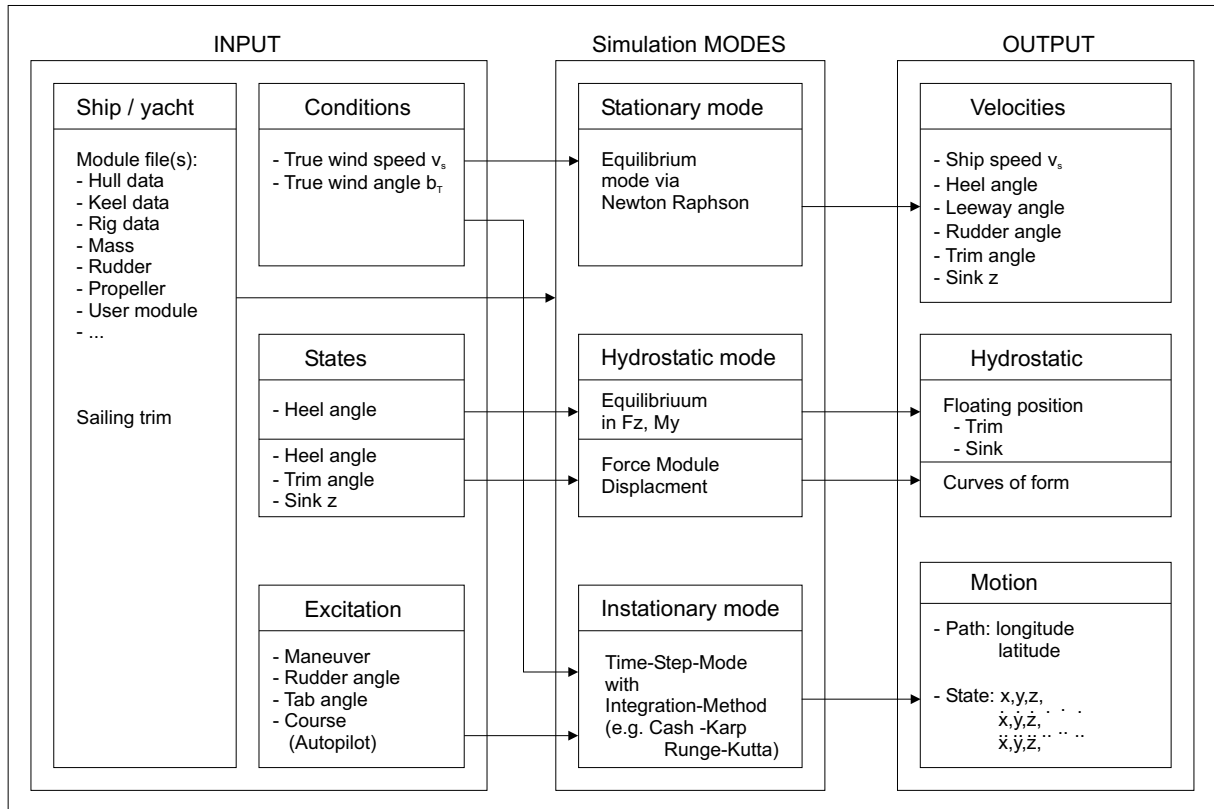


Fig. 3: Modes within FRIENDSHIP-Equilibrium

module and are then considered as part of the excitation force. A module for linear coefficients of added mass and damping is readily available. Alternatively, an additional module which describes these forces can be defined by the user. Nonlinear coefficients as used by *Masayuma et al.* (1995) for instance, may be implemented in more advanced modules. The accelerations are calculated by solving the equation of motions via a time-step method with a chosen integration routine. FRIENDSHIP-Equilibrium currently makes available a fourth-order Runge-Kutta scheme, a fifth-order Runge-Kutta-Feldberg scheme as well as a fifth-order Cash-Karp Runge-Kutta variable time step scheme. In the variable time step integration the time step is adjusted so that the difference of the result using a fourth order Runge-Kutta scheme and the result using a fifth order scheme is less than a selected tolerance for each of the state variables. A time scale can be set such that the maneuver is executed in real time or a specified fraction of it that. The process flow of the time-stepping procedure is sketched in Fig. 4.

The velocity and acceleration are combined into one state vector:

$$\underline{s} = (x, y, z, \phi, \theta, \psi, \dot{x}, \dot{y}, \dot{z}, \dot{\phi}, \dot{\theta}, \dot{\psi}, \ddot{x}, \ddot{y}, \ddot{z}, \ddot{\phi}, \ddot{\theta}, \ddot{\psi})^T . \quad (1)$$

All state variables are computed as functions of time. The trajectory is displayed in global coordinates. Moreover, any desired state variable can be plotted during the operation.

3 Mathematical model

3.1 Coordinate systems

The coordinate systems used are displayed in Fig. 5. All systems are right handed and orthogonal. The forces are calculated in absolute coordinates **A** which is a partially body fixed moving coordinate system.

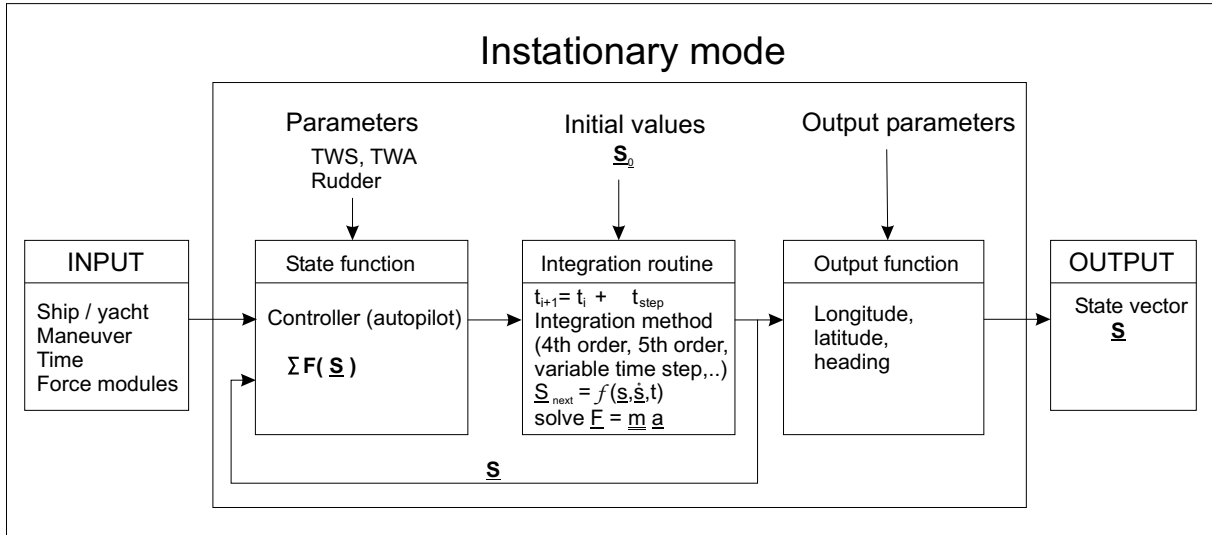


Fig. 4: Maneuvering simulation within FRIENDSHIP-Equilibrium

The x-axis is directed along the vessel's centerline while the z-axis is perpendicular to the undisturbed free surface. The heel and trim angles are determined via the rotation between the body fixed coordinate system \mathbf{B} and the absolute coordinates \mathbf{A} . The hydrodynamic coordinate system \mathbf{H} is a moving coordinate system directed along the track to determine the leeway angle. The boat's track is shown in the global coordinate system which represents the earth-bound coordinates. The location of the origin must be body fixed but can be arbitrarily selected, e.g. the center of gravity. The transformation of the forces from a body fixed system \mathbf{B} to the absolute coordinate system \mathbf{A} is accomplished by left multiplication with the transformation matrix

$$\underline{T}_{BA} = \begin{bmatrix} \cos \theta & \sin \phi \sin \theta & \cos \phi \sin \theta \\ 0 & \cos \phi & -\sin \phi \\ -\sin \theta & \sin \phi \cos \theta & \cos \phi \cos \theta \end{bmatrix} \quad (2)$$

for arbitrary heel and pitch angles ϕ and θ , respectively.

3.2 Specification of ship motions

The ship motions comprises in six degrees of freedom – three translations and three rotations. These motions are described in the absolute coordinate system illustrated in Fig. 6.

Table III: Generalized displacements of a vessel

| | | |
|----------|---------------------------------------|-------|
| x | Translation in longitudinal direction | surge |
| y | Translation in transversal direction | sway |
| z | Translation in vertical direction | heave |
| ϕ | Rotation about the longitudinal axis | roll |
| θ | Rotation about the transversal axis | pitch |
| ψ | Rotation about the vertical axis | yaw |

The state variables are expressed in vector form:

$$\underline{x} = (x, y, z, \phi, \theta, \psi)^T ,$$

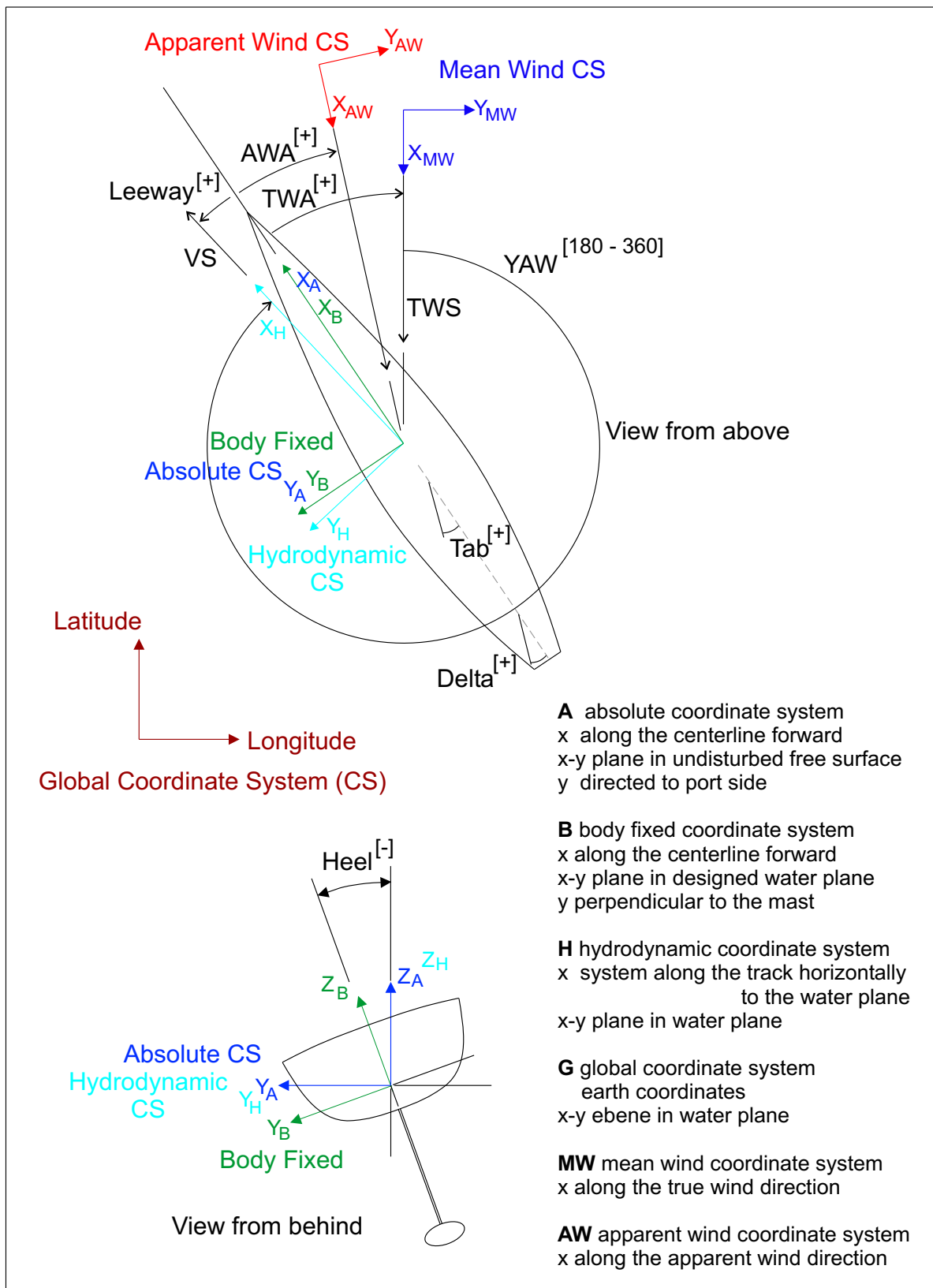


Fig. 5: Coordinate systems used in FRIENDSHIP-Equilibrium

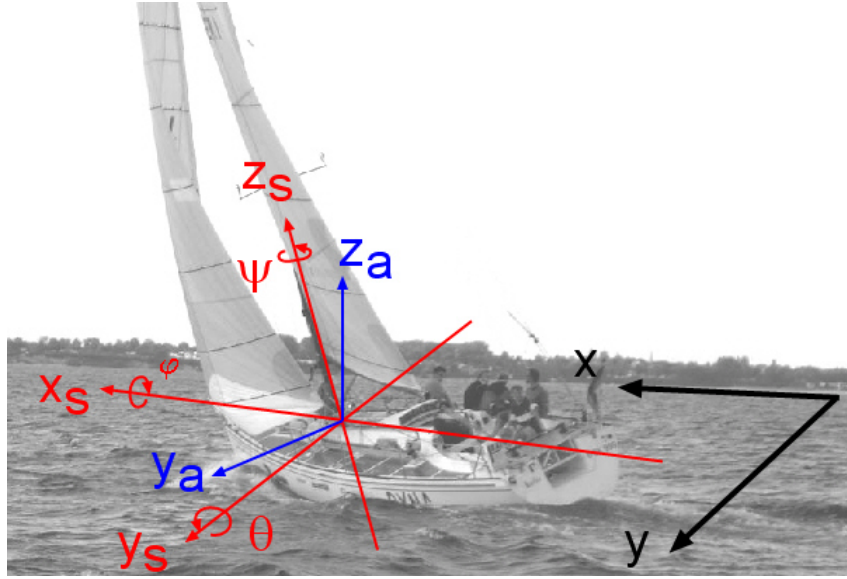


Fig. 6: Definition of ship coordinate systems in six degrees of freedom

$$\underline{\dot{x}} = (\dot{x}, \dot{y}, \dot{z}, \dot{\phi}, \dot{\theta}, \dot{\psi})^T ,$$

$$\underline{\ddot{x}} = (\ddot{x}, \ddot{y}, \ddot{z}, \ddot{\phi}, \ddot{\theta}, \ddot{\psi})^T .$$

3.3 Equations of motion

According to Newton's second law the motion of a rigid body in the earth-bound coordinate system can be expressed as follows:

$$\underline{F}_{external} = \underline{\underline{M}} \cdot \underline{\ddot{x}} \quad (3)$$

The forces and moments are combined in a generalized force vector

$$\underline{F} = (F_x, F_y, F_z, M_x, M_y, M_z)^T$$

while the accelerations are given in the state vector

$$\underline{\ddot{x}} = (\ddot{x}, \ddot{y}, \ddot{z}, \ddot{\phi}, \ddot{\theta}, \ddot{\psi})^T .$$

The mass matrix $\underline{\underline{M}}$ then contains the following components:

$$\underline{\underline{M}} = \begin{pmatrix} m & 0 & 0 & 0 & m \cdot z_{CG} & -m \cdot y_{CG} \\ 0 & m & 0 & -m \cdot z_{CG} & 0 & m \cdot x_{CG} \\ 0 & 0 & m & m \cdot y_{CG} & -m \cdot x_{CG} & 0 \\ 0 & -m \cdot z_{CG} & m \cdot y_{CG} & I_{xx} & -I_{xy} & -I_{xz} \\ m \cdot z_{CG} & 0 & -m \cdot x_{CG} & -I_{yx} & I_{yy} & -I_{yz} \\ -m \cdot y_{CG} & m \cdot x_{CG} & 0 & -I_{zx} & -I_{zy} & I_{zz} \end{pmatrix}$$

with $m = \rho \cdot \nabla$ for steady conditions.

A linear force system is assumed to specify the acting forces on the vessel. The hydrodynamic forces are divided into their translation, velocity and acceleration components.

$$\underline{F}_{Hydrodynamic} = -\underline{A} \cdot \ddot{x} - \underline{B} \cdot \dot{x} - \underline{C} \cdot x \quad (4)$$

The matrix \underline{A} includes the added mass coefficients and \underline{B} refers to the damping coefficients. Hydrostatic forces are included in the third term. Velocity and acceleration depend on the reference system. Therefore, the centripetal acceleration \underline{a}_{cp} must be considered in the accelerated absolute coordinate system, see e.g. *Gummert and Reckling* (1985). Forces resulting from wind and waves and all other additional external forces are combined in so-called further environmental forces $\underline{F}_{Environmental}$ to be determined by the appropriate modules of the program.

The following equations are solved by the integration routine:

$$(\underline{M} + \underline{A}) \cdot (\ddot{x} - \underline{a}_{cp}) = -\underline{B} \cdot \dot{x} - \underline{C} \cdot x + \underline{F}_{Environmental} \quad (5)$$

3.4 Coefficients

In order to solve the equations of motion the matrices of the added mass and damping coefficients must be determined. In general, several methods are available to identify these coefficients:

- Estimation of the coefficients from simple shapes
- Formulas developed by Clarke
- Lewis transformation applying conformal mapping and strip theory
- Inverse Fourier transformation of frequency dependent coefficients
- Viscous flow simulations
- Model experiments
- (Full scale measurements)

The order in this list roughly corresponds the associated effort, the quality of the outcome being proportional. For a first estimation of the added mass the formulas for simple shapes like cylinders and ellipsoids as published by *Saunders* (1957) can be utilized. Clarke, see *Lewis* (1988), developed formulas depending on the ship's geometry to evaluate linear coefficients for added mass and damping. More accurate values of the coefficients can be derived from conformal mapping of each section by applying Lewis transformation and utilizing strip theory. Here the three-dimensional flow problem is solved via the integration of two-dimensional sub-domains along the hull. This method is by definition only valid for slender bodies, but reasonable values can be obtained. *Bohlmann* (1990) for instance used this theory to identify the coefficients for submarines and provided formulas to calculate the damping coefficients. The hydrodynamic mass in x-direction can be calculated from the the added mass for an ellipsoid given in *Bertram* (2000). However, this yields only rough estimates for typical sailing yacht bow and stern shapes.

Frequency dependent coefficients can be obtained from seakeeping calculations. A routine to transform these coefficients with an inverse Fourier transformation from frequency-domain to time-domain in order to identify the coefficients in the state space (*Schmiechen*, 1973) has been studied by *Richardt* (2004) for implementation into FRIENDSHIP-Equilibrium. Finally, the determination of the coefficients by model testing as carried out by *Wolff* (1981) for example is recommended, of course, but expensive with regard to both time and money.

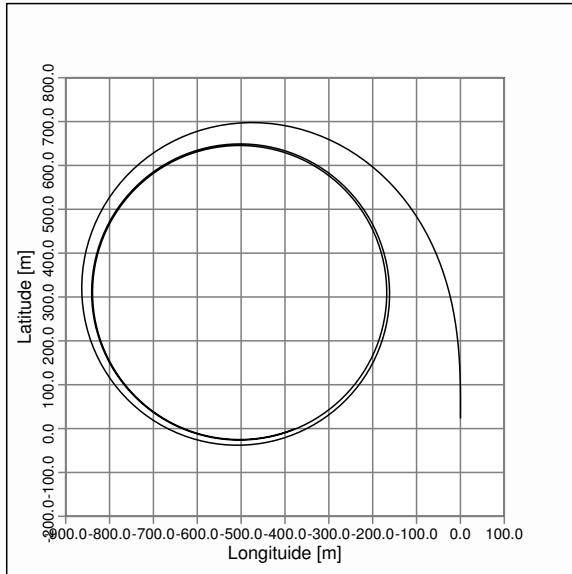


Fig. 7: Turning test for cargo ship

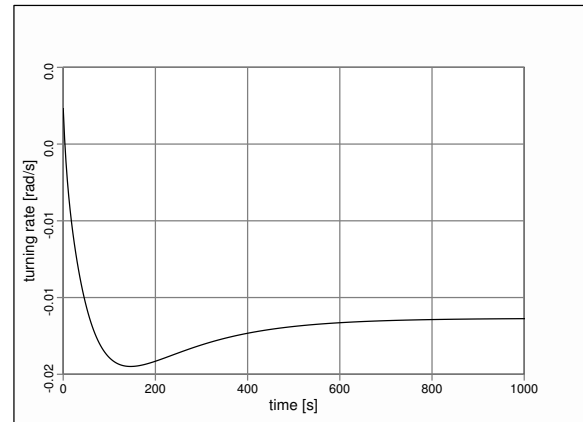


Fig. 8: Turning rate ψ during turning test

4 Simulations

Maneuvering simulations were processed for a representative cargo ship and a typical sailing yacht. For the former a Mariner ship was selected which is a common test case. For the latter the full scale research yacht DYNA was chosen. DYNA resembles a 10m IMS sailing yacht and was developed at the Technical University Berlin, *Hochkirch* (2000). Full scale experiment data are available for this yacht which can be used for validation purposes.

4.1 Maneuver simulations for a cargo ship

For the force calculations of the Mariner ship a simple module which includes the estimation of the propeller thrust, the hull resistance and the rudder forces was integrated. The main parameters were taken from *Wolff* (1981). The coefficients for added mass and damping were determined by the formulas of *Clarke* as published in *Lewis* (1988). For the maneuverability analysis various common test maneuvers are implemented. The standard maneuvers like turning and zig zag tests are readily available as maneuvering modules. The rudder angle and the maximum course angle are free to be specified. The fourth-order Runge-Kutta integration method with a time step of 0.2 seconds was applied for these simulations. The calculated track for a turning test with a rudder angle of 20° is displayed in Fig. 7. Fig. 8 shows the turning rate during this simulation. After 1000 seconds a constant turning rate was reached. The tactical diameter of 670 meters as determined from Fig. 7 is met after 10 minutes.

The zig zag test is typically executed to investigate turning capabilities. Different rudder angles and maximum course angles can be used. The track for a zig zag test with a rudder and a maximum course angle of 20° is displayed in Fig. 9. For lack of space, the longitude and latitude axis are scaled 2.5:1.

4.2 Velocity prediction for a sailing yacht

In a second example the FRIENDSHIP-Equilibrium was used for the velocity prediction of a 10m IMS yacht. Various modules have been developed from research results for that yacht, see *Hochkirch* (2000). The forces acting on keel and rudder were defined in the modules called 'Keel' and 'Rudder'. The

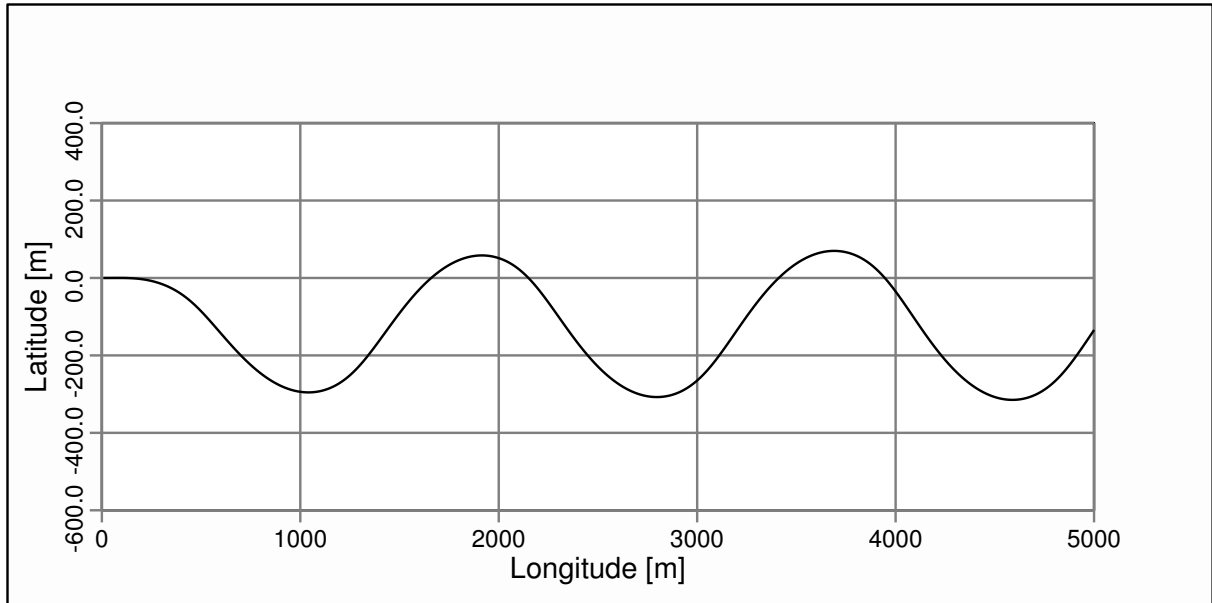


Fig. 9: Zig zag test for cargo ship (scaled 2.5:1)

aerodynamic rig forces were calculated with the module 'GenerigRig'. Different modules were used for displacement, mass, crew and windage. So as to demonstrate the application of trim parameters a set of different trim parameters was included in the 'GenerigRig' module. These parameters are explained in Table IV.

Table IV: Example of trim parameters in the module 'GenerigRig'

| | | | |
|----------|-------|------------------------|---|
| σ | reef | $V_S \rightarrow \max$ | reefing of the sails to be adjusted so as to gain optimum speed |
| τ | flat | $V_S \rightarrow \max$ | flatening of the sails to be adjusted so as to gain optimum speed |
| t | twist | $V_S \rightarrow \max$ | reduction of the center of effort of the aerodynamic forces so as to improve the overall performance of the boat when sailing upwind, see <i>Jackson (2001)</i> for details |

The offsets of the hull and geometric data are given in the input file. Parameters like water density, gravity etc. can be defined in either the input file or in the program. Before calculating the equilibrium for sailing conditions the model must be in hydrostatic balance. Therefore, the equilibrium for trim and sinkage in rest position is determined first. The states to be calculated can be defined in the 'Cycle Range' window as indicated in Fig. 2. In the example the true wind velocity VWT is considered from 4 m/s to 9 m/s for all courses between an angle of 35° to 180° with an increment of 5°. In the output table the velocities for the equilibrium of all states is listed as pictured in Fig. 10. In this figure a polar diagram is also shown for the six wind speeds.

4.3 Maneuver simulations for a sailing yacht

For the instationary calculations an additional module was defined to compute the added mass and damping. The module was called 'AddedMass' and the coefficients were determined by means of Lewis transformation as explained in section 3.4. The dynamic changes of the effective angles at the keel, rudder and the sails during the motions are accounted for in the lift calculations. The conditions were chosen to meet the situation encountered during full-scale measurements. The tacking simulations were processed with a true wind velocity of 9.59 m/s. The close-hauled courses were steered at 37° to the wind. The rudder angle was automatically adjusted by the autopilot. The factors of the PID controller can be adapted to

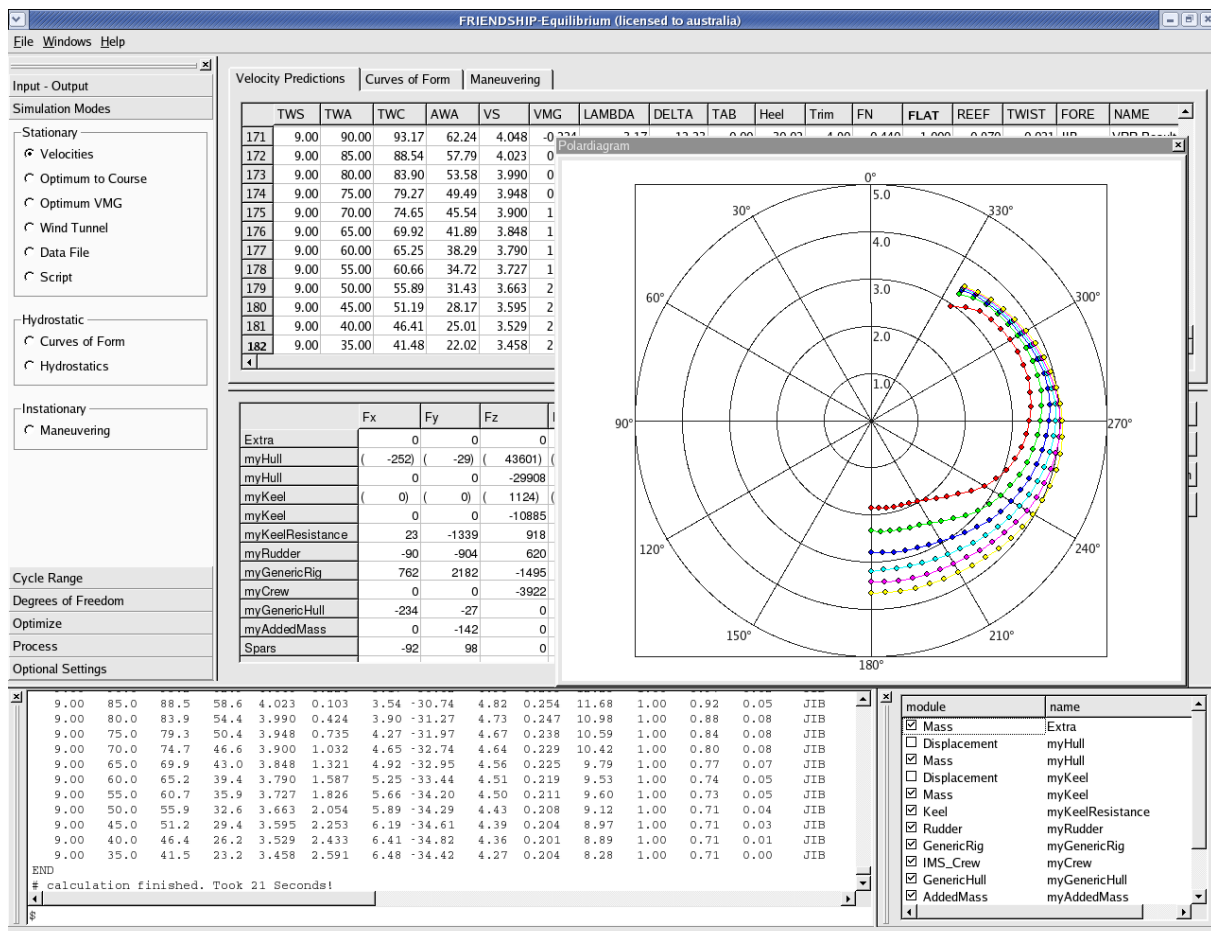


Fig. 10: VPP for a sailing yacht

the problem and consequently influence the course keeping capabilities. Tacks can also be simulated in manual maneuvers which enables the users of the program to steer themselves by means of a joystick. A tacking maneuver in which the rudder is linked to a maximum angle until a certain course angle on the other tack is reached can be applied. The fifth-order Cash-Karp integration routine was used to solve the equations of motions.

Fig. 11 depicts the FRIENDSHIP-Equilibrium running in the maneuvering mode. The settings of the autopilot can be seen in the 'ManeuverEditor' on the upper left. Several graphs are integrated in the 'Maneuvering' window. On the left the velocity as function on time is displayed. In the other graphs the track and the heeling angle are plotted. Naturally, the quantities to be shown can be selected as needed.

The results of the tacking simulations are compared to data of the full scale measurements. The yaw angle is displayed in Fig. 12 while the associated heel angle is shown in Fig. 13. The oscillation of the full-scale data was caused by heavy seas and strong unsteady winds during the measurements. Nevertheless, a rather good correlation between the simulated and the measured data can be observed. From the starting course to head wind the tack needs 7 seconds. A stable heel angle is achieved after 43 seconds and the new course is fixed after about one minute. In real life these results depend on the helmsman. In the simulations the value may be optimized by adjusting the autopilot. The optimum speed is reached after about 82 seconds. The sail trim was idealized during the simulated tack. Naturally, this typically is not the case in reality. Therefore, additional parameters will have to be integrated in the future so as to take into account the time needed by the crew to trim the sails optimally.

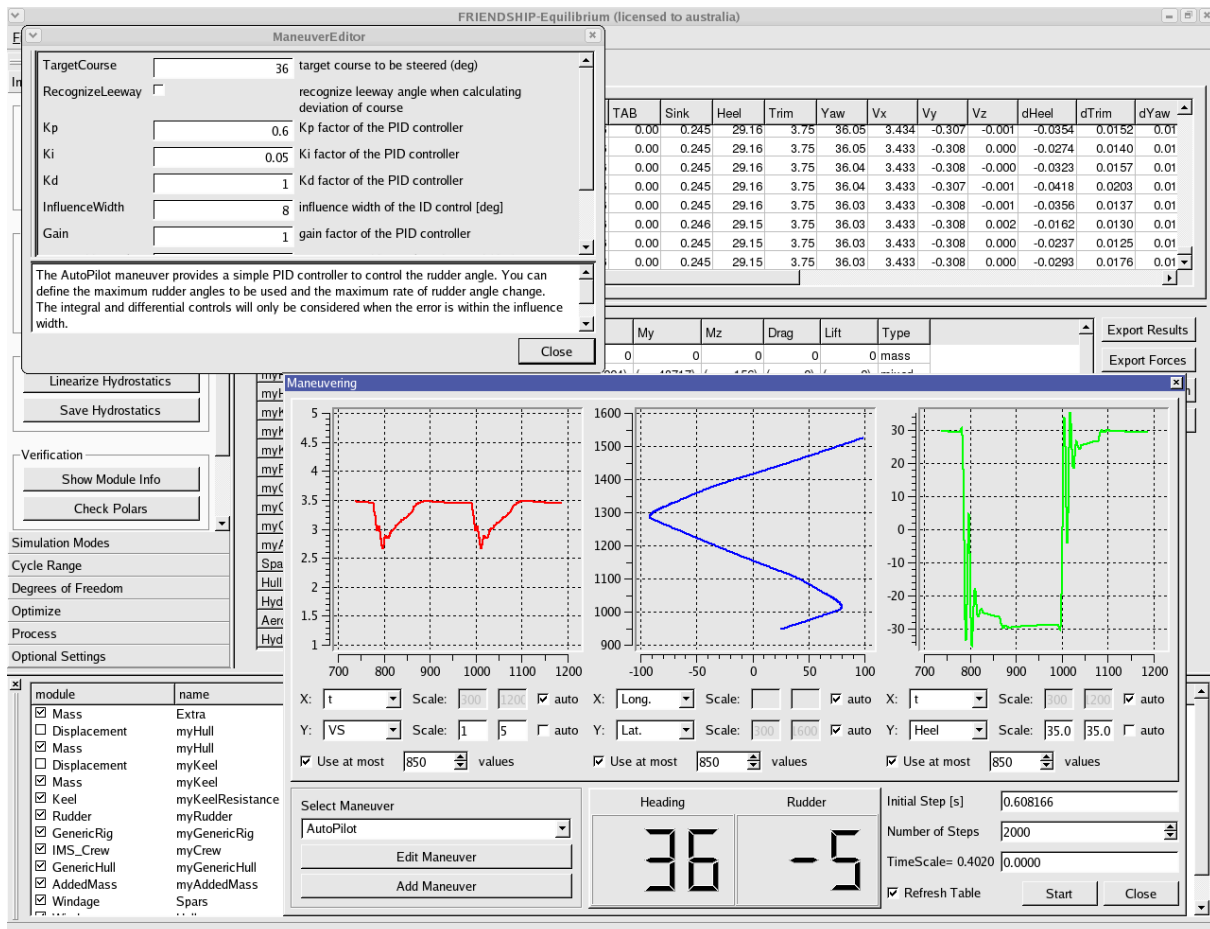


Fig. 11: Maneuvering simulation for a 10m sailing yacht

5 Conclusion

FRIENDSHIP-Equilibrium is an open modular workbench which follows a force approach. All forces and moments are defined in separate modules. A wide range of force modules is readily available within the software system like hydrostatics, mass, hydrodynamics from model tests, added resistance in waves, Delft series for resistance of sailing yachts and keels (*Keuning and Sonnenberg, 1999*), lifting surfaces such as keels and rudder, IMS type rig models and many more. Additional modules to accommodate individual forces and scenarios can be easily integrated for specific simulations.

FRIENDSHIP-Equilibrium supports stationary, hydrostatic and instationary simulation modes. The accuracy of the results, naturally depends on the implementation of the underlying mathematical model used in the force modules. For instationary conditions reasonable estimations of added mass and damping terms are of importance since the simulations are sensitive to these forces. An accurate identification of this terms is required for realistic results.

Special focus was given on instationary simulations for which examples were presented for two types of vessels – a cargo ship and a sailing yacht – to show the applicability and the potential usage of the system. The sailing yacht maneuver presented was the tacking of a 10m IMS yacht. It comprised a comparison between the simulations based on FRIENDSHIP-Equilibrium and experimental data from full-scale sailing tests which showed very promising correlation.

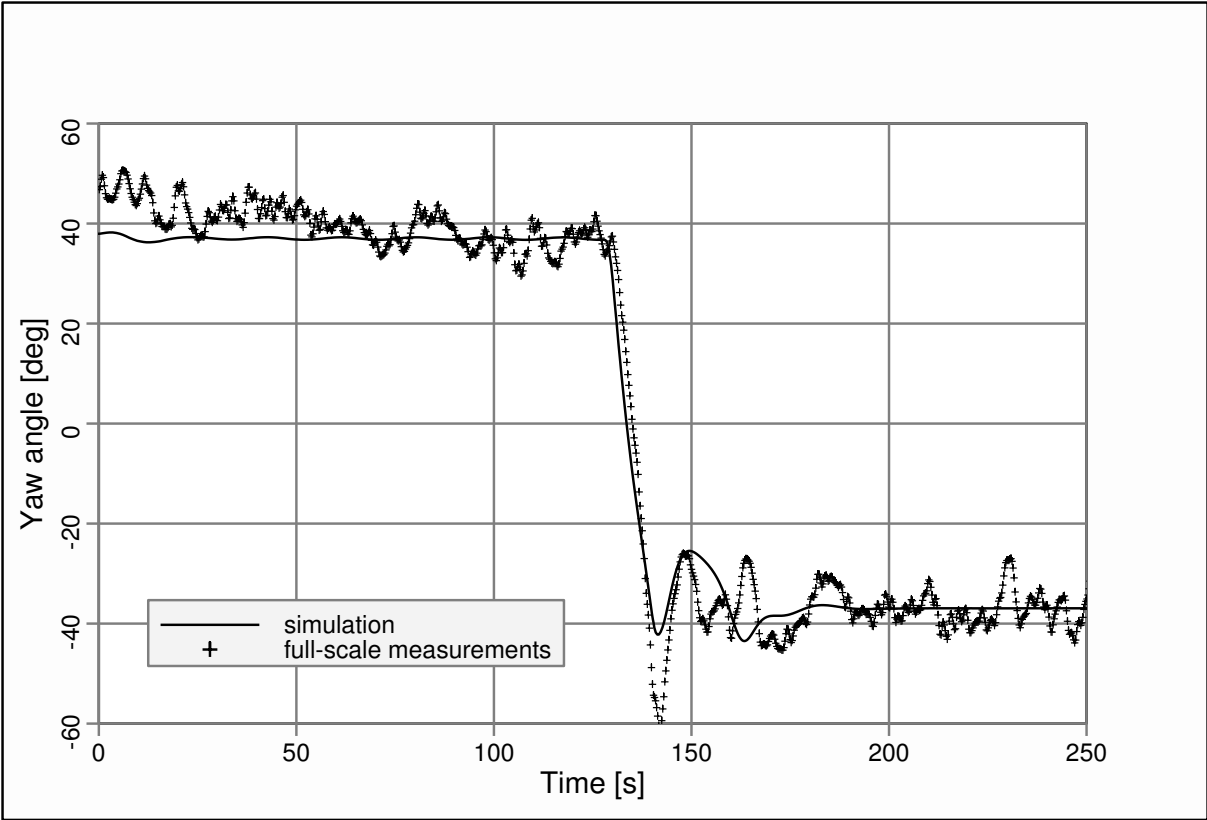


Fig. 12: Yaw angle

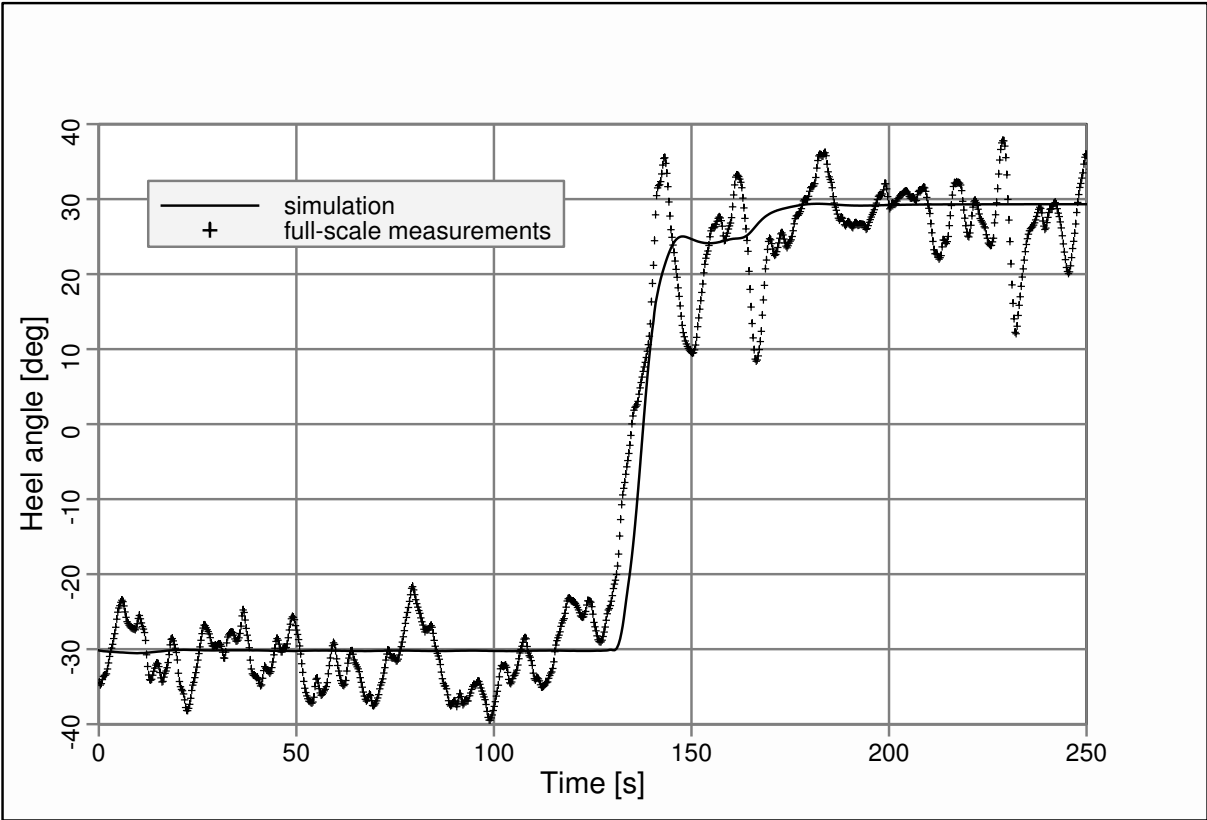


Fig. 13: Heel angle

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